Highway on-ramp control

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We study the phase transition on a highway induced by the fluctuations of on-ramp flow. The highway traffic is provided by a hydrodynamical model. We analyze the characteristics of perturbations to induce congestion near on ramp. The phase boundary is obtained. A scaling relation is revealed. We also analyze the time evolution of the local density profile. Conventional control mechanisms to regulate the on-ramp flow are examined. A control scheme is proposed to suppress the congestion.

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I. INTRODUCTION

The study of traffic flow near on ramps of a highway system has revealed a rich spectrum of phenomena and attracted research interests from physicists recently [1–5]. On the section of highway without ramps, the well formed traffic jams always propagate in the upstream direction; while near an on ramp, a new type of congestion appears. The traffic jams seem to be localized. They move back and forth around the ramp with the same structure. Or the congestion may extend with time, but always have one end fixed at the ramp. Or they may even stay motionless. The current theoretical research is then focused on the characteristics of this new type of congestion [6–10]. To physicists, the highway traffic is basically a one-dimensional many-body system with strong correlations among vehicles. The system is driven far from equilibrium, where the steady states are characterized by nonvanished flows. The congestion is taken as a nontrivial phase transition resulted from the instability of the system. In the paper, we study the hysteretic phase transitions induced by the fluctuations of on-ramp flow. The highway traffic is simulated by a well calibrated hydrodynamic model [11–13]. The resulting phase diagram is presented in the following section. The characteristic of the fluctuations is analyzed. In Sec. III, we study the time evolution of density profile in the emergence of congestion. Various control schemes aimed to suppress the congestion are discussed in the final section.

II. PHASE DIAGRAM

In the hydrodynamic model of highway traffic, the system is described by the following partial differential equations:

$$\rho\left(\frac{\partial v}{\partial t}+v\,\frac{\partial v}{\partial x}\right)=\frac{\rho}{\tau}[V(\rho)-v]-c_0^2\frac{\partial \rho}{\partial x}+\mu\,\frac{\partial^2 v}{\partial x^2}, \quad (1)$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho v)=q_i(t)\;\phi(x). \quad (2)$$

These two equations are analogous to the Navier-Stokes equation and the continuity equation, respectively. The highway traffic is described by the local density $\rho(x,t)$ and the local velocity $v(x,t)$. The traffic flow is understood as $q(x,t)=\rho(x,t)\,v(x,t)$. The intrinsic properties on the main highway is prescribed by the safe-velocity functional $V(\rho)$ and three parameters, τ, c_0, and μ, which are constants related to the effects of relaxation, anticipation, and dampening, respectively [11–13]. The on-ramp flow is denoted by $q_i(t)$ with a spatial distribution $\phi(x)$ localized at $x=0$. In this work [14], we perform extensive numerical simulations over a system of 60-km highway with an on-ramp right in the middle (position $x=0$). The open boundary conditions are employed. The values at the upstream boundary are kept fixed; the values at the downstream boundary are obtained by linear extrapolation from the neighborhood.

When the vehicle density is small, the highway traffic is a homogeneous flow. In the upstream ($x\ll 0$), the safe velocity is achieved and the flow $q(x,t)$ is a constant denoted by q_{up}. The on-ramp flow q_{rmp} provides a small transition layer near the ramp, which transforms the upstream flow into the downstream flow. In the downstream ($x\gg 0$), the safe velocity is also achieved and the flow is simply $(q_{up}+q_{rmp})$. The system is then specified by two parameters the upstream flow q_{up} and the on-ramp flow q_{rmp} [17,18].

When the vehicle density increases, the congestion becomes one of the stable solutions. The transitions between the homogeneous flow and the congestion can be triggered by fluctuations of the on-ramp flow. We study the criterion of perturbations to induce such transitions. The system is prepared to start with a homogeneous flow specified by two parameters q_{up} and q_{rmp}. To simulate the inflow of a large number of vehicles within a short period, the perturbations are prescribed in the following form:

$$q_i(t)=\begin{cases} q_{rmp}+\Delta q & t_0<t<t_0+\Delta t \\ q_{rmp} & \text{otherwise.}\end{cases} \quad (3)$$

The perturbations are characterized by two parameters Δq and Δt, which specified, respectively, the height and duration of the extra on-ramp flow. The initial time t_0 is irrelevant as we start from a stationary state.

The induced phase transitions from the homogeneous flow to the congestion can be observed within a certain range of (q_{up},q_{rmp}). The phase diagram is shown in Fig. 1. When $q_{up}+1.2\,q_{rmp}<2060$ vehicles per hour (veh/h), the homogeneous flow is the only stable solution. The perturbations always decay with time. When $q_{up}>2060$ veh/h, the congestion will not be localized to the ramp. The effects of an on ramp become insignificant. When $q_{up}+q_{rmp}>2300$ veh/h, the homogeneous flow becomes unstable. The congestion

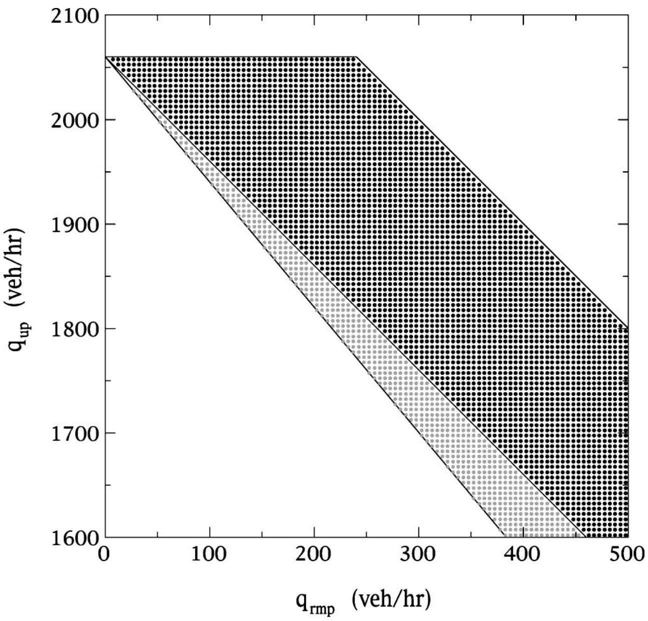

FIG. 1. Phase diagram in the (q_{up},q_{rmp}) plane. The scaling region is shown by black dots, the nonscaling region is shown by the gray dots.

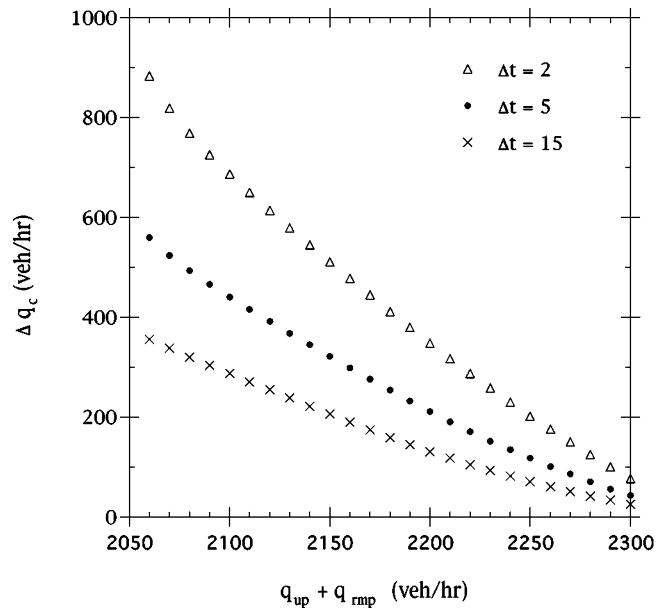

FIG. 3. Critical extra flow Δq_c as a function of downstream flow $(q_{up}+q_{rmp})$ for various values of Δt. The parameter $q_{up} = 1800$ veh/h.

can be induced by a very small perturbation. Within these boundaries, the transitions can be induced when the perturbations are large enough, i.e., with a given Δt, a low bound of Δq is required to trigger the transition. This critical value is denoted by Δq_c, which depends on q_{up}, q_{rmp}, and Δt. We further observe that within the region $2060 < q_{up} + q_{rmp} < 2300$ veh/h and $q_{up} < 2060$ veh/h, the critical value Δq_c is scaled with the downstream flow $(q_{up}+q_{rmp})$. That is, Δq_c depends only on Δt and the combination $(q_{up}+q_{rmp})$.

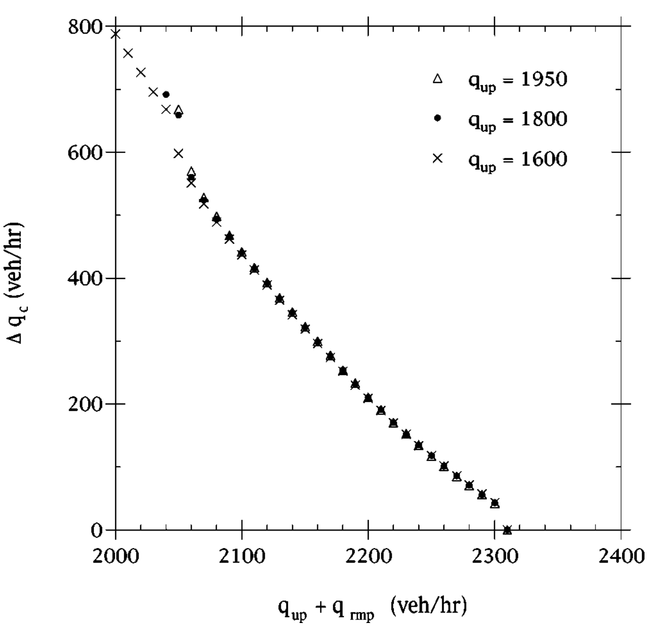

FIG. 2. Critical extra flow Δq_c as a function of downstream flow $(q_{up}+q_{rmp})$ for various values of q_{up}. The parameter $\Delta t = 5$ min.

The results are shown in Fig. 2. The critical height Δq_c decreases with the increase of both the downstream flow $(q_{up}+q_{rmp})$ and the duration Δt. The results are shown in Fig. 3. It is also observed that Δq_c scales approximately with $\sqrt{\Delta t}$ at a fixed $(q_{up}+q_{rmp})$. Thus, the quantity $(\Delta q_c \sqrt{\Delta t})$ scales approximately with $(q_{up}+q_{rmp})$.

Outside the scaling region, the phase transitions can still be observed within the region of $q_{up}+q_{rmp} > 2060$ veh/h and $q_{up} + 1.2\, q_{rmp} < 2060$ veh/h. Such small region can also be expressed as $q_{rmp} < 2060 - q_{up} < 1.2\, q_{rmp}$. In such cases, the critical value Δq_c depends on q_{up} and q_{rmp} separately. When q_{up} approaches 2060 veh/h, the phase transitions can only be triggered with a small range of q_{rmp}. As q_{up} decreases, a much wider range of q_{rmp} is supported. As the downstream flow is less than in the scaling region, the congestion can only be induced by a Δq_c larger than in the scaling region, see Fig. 2.

The two flows 2060 and 2300 veh/h seem to be special. When the on ramp is closed, i.e., $q_{in}(t) = 0$, the traffic flow on the main highway is often differentiated by three special values, 2060, 2300, and 2340 veh/h. The maximum flow can be achieved in the homogeneous flow is 2340 veh/h. However, the homogeneous flow becomes unstable when the flow is larger than 2300 veh/h. Furthermore, the homogeneous flow is stable to large perturbations only when the flow is less than 2060 veh/h. When $q > 2060$ veh/h, the congestion begins to emerge and a well formed traffic jam always moves to the upstream with a constant velocity. Thus, for $q < 2060$ veh/h, the homogeneous flow is the only stable traffic state. For $q > 2300$ veh/h, the traffic jam becomes the only stable traffic state. For $2060 < q < 2300$ veh/h, both the homogeneous flow and the congestion are stable. The transition between these two states can be induced by external perturbations.

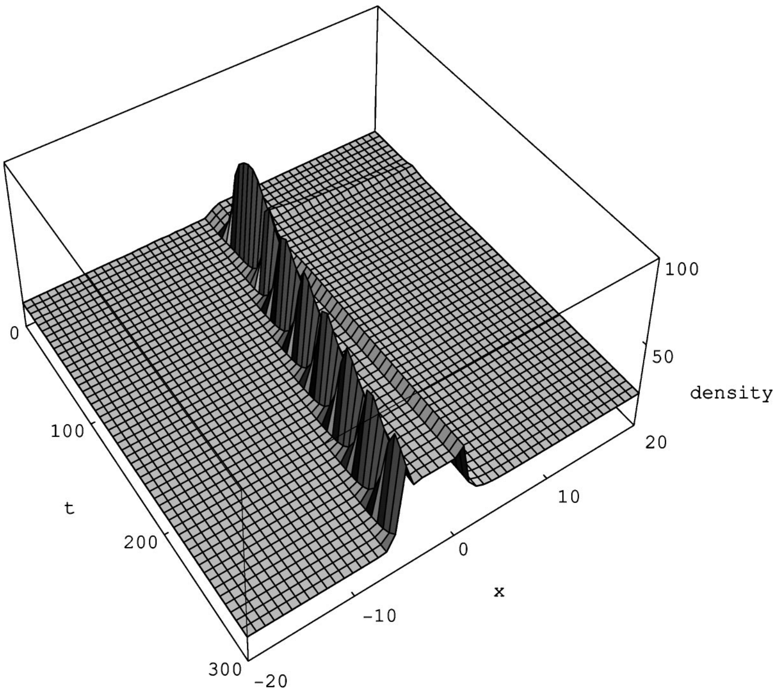

FIG. 4. Typical density profile $\rho(x,t)$ in the nonscaling region. The parameters: $q_{up} = 1600$ veh/h, $q_{rmp} = 440$ veh/h, $\Delta q = 670$ veh/h, $\Delta t = 5$ min, and $t_0 = 30$ min.

With the on ramp, the behavior of the system can be deduced. In the scaling region, the instability of the system is determined by the downstream flow. The metastability is observed when the flow is between 2060 and 2300 veh/h. As to the congestion in the nonscaling region, it can be taken as the manifestation of boundary-induced phase transitions [19].

III. DENSITY PROFILE

Next, we consider the time evolution of density profiles in the emergence of congestion. The congestion near the on ramp is caused by the inflow of a large amount of vehicles within a short period. With naive expectation, the appearance of traffic jams is attributed to the incapability of the main highway in dissipating the extra flow from the on ramp. The extra on-ramp flow and the congestion are then expected to appear simultaneously. However, this naive expectation is valid only within the small range of nonscaling region, i.e., $q_{up} + q_{rmp} < 2060$ veh/h and $q_{up} + 1.2\ q_{rmp} > 2060$ veh/h. The lower bound of the extra flow Δq_c is quite large in these cases. When q_{up} is large, the congestion appears to be stationary; when q_{up} is small, the congestion expands to the upstream as time evolves. The typical results are shown in Fig. 4.

In the scaling region, i.e., $2060 < q_{up} + q_{rmp} < 2300$ veh/h, the congestion can be induced by a much smaller Δq_c. In such cases, quite contrary to the naive expectation, the traffic jams appear much later in time than that of the extra inflow. Delay of more than 1 h is often observed. The typical results are shown in Fig. 5. Right after the extra inflow, a small traffic jam appears and propagates away from the ramp to the downstream. The traffic jam enlarges slowly and the speed of propagation is decreasing. At a certain time, the traffic jam turns around and propagates to the upstream, i.e., moving back to the on ramp. The traffic jam enlarges quickly and a huge congestion is induced when it moves across the on ramp. When the extra flow is only slightly larger than Δq_c, the small traffic jam can propagate a long distance before its turning around. The appearance of the congestion is then delayed to a much later time. In contrast, when the extra flow is less than Δq_c, the phase transition will not be induced. We can still observe the small traffic jam moves away from the ramp to the downstream. In such cases, the traffic jam propagates with a slowly increasing speed but decreasing amplitude. And it just disappears in a later time. The q_{up} dependence is similar to the cases of the nonscaling region. When q_{up} is large, the congestion is observed to oscillate back and forth around the ramp; when q_{up} is small, the congestion expands to the upstream as time evolves.

In the nonscaling region, a well formed traffic jam cannot be sustained in the main highway, neither in the upstream nor in the downstream. The congestion can only emerge right at the ramp. In the scaling region, the traffic jams still cannot be sustained in the upstream; while they can be developed in the downstream. The congestion at the ramp can be related to a well formed traffic jam in the downstream. Thus, the same phase transition can also be triggered by a traffic jam caused by fluctuations in the downstream. As the traffic flow further increases, the traffic jams can also be developed in the upstream when $q_{up} > 2060$ veh/h. Then the congestion will no longer be confined to the vicinity of the on ramp. The traffic jams developed in the downstream always move to the upstream. They just move across the on ramp smoothly without inducing significant effects. The congestion caused by the extra inflow from on ramp will soon move away from the ramp to the upstream.

IV. DISCUSSIONS

In this paper, we study the emergence of congestion induced by fluctuations of the inflow through the on ramp. The

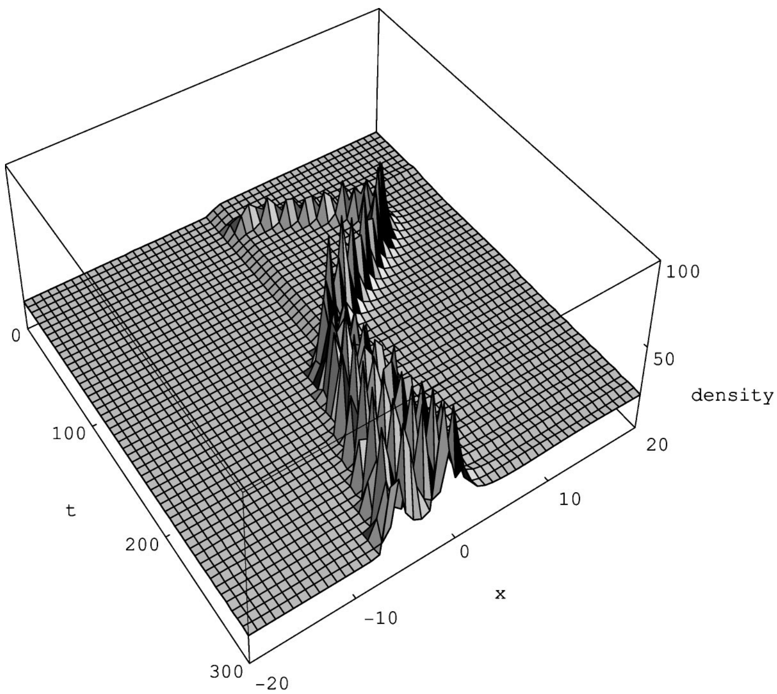

FIG. 5. Typical density profile $\rho(x,t)$ in the scaling region. The parameters: $q_{up} = 1800$ veh/h, $q_{rmp} = 310$ veh/h, $\Delta q = 420$ veh/h, $\Delta t = 5$ min, and $t_0 = 30$ min.

congestion is not caused by random fluctuations. A definite type of perturbations is required to trigger the transitions. An extra flow within a short time is necessary, which is characterized by two parameters, Δq and Δt. To induce the congestion, the extra flow must be larger than the critical value Δq_c, which then depends on q_{up}, q_{rmp}, and Δt. It is interesting to note that the criterion to a phase transition is not associated directly with the number of vehicles in the extra flow, which is equal to $(\Delta q_c \ \Delta t)$. We observe that the quantity $(\Delta q_c \sqrt{\Delta t})$ scales approximately with the downstream flow $(q_{up} + q_{rmp})$. At the same downstream flow, a larger number of vehicles is required when the perturbation has a longer duration. The scaling also implies that the same extra flow will induce the congestion at a smaller q_{rmp} when q_{up} is larger.

The congestion is induced, not spontaneously emerged. Within a wide range of the flows, both the free flow and the congestion are stable. Thus appropriate control mechanisms can be employed to suppress the traffic jams. From the phase diagram of Fig. 1, the highway is free of congestion when $q_{up} + 1.2\, q_{rmp} < 2060$ veh/h. With a larger upstream flow q_{up}, the on-ramp flow q_{rmp} should be restricted to a smaller value. The conventional on-ramp control is carried out by setting up a traffic light at the ramp before the intersection

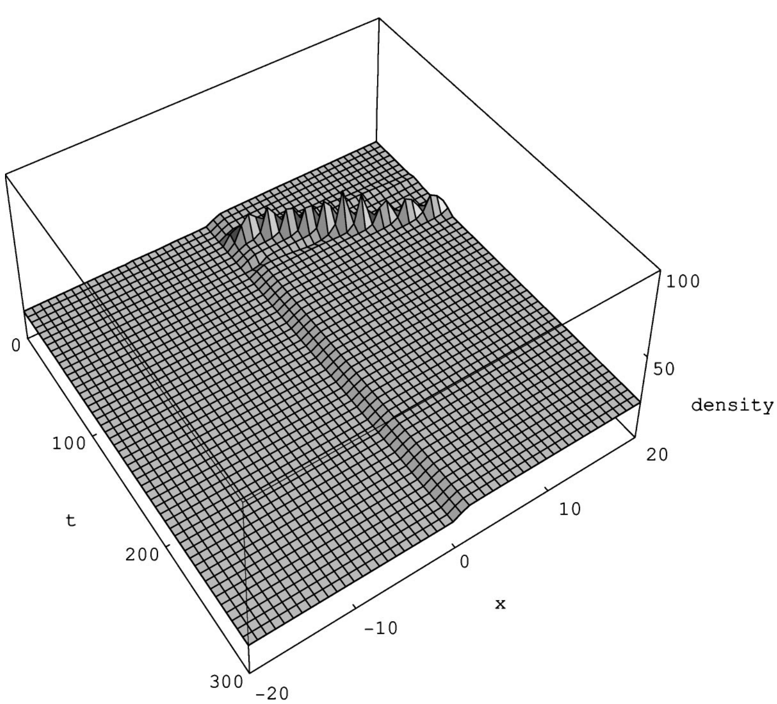

FIG. 6. Same as Fig. 5 except that the on ramp is closed for 1 min at $t = 60$ min.

with the main highway. The traffic light switches periodically to red and green. By allocating different time to red and green phases, the average value of the on-ramp flow can be easily restricted. With the operation of the traffic light, the phase diagram is the same as shown in Fig. 1, with the parameter q_{rmp} now denotes the average value of the on-ramp flow. In the nonscaling region, the phase transitions must be triggered with a very large value of Δq_c. It is correct in equation mathematically, but impractical in reality. If we consider only the phase transitions in the scaling region, a softer restriction of $q_{up}+q_{rmp}<2060$ veh/h will guarantee the highway free from the congestion. The operation of the traffic light should depend on the upstream flow. With the increase of q_{up}, one should decrease q_{rmp} by allocating more time to the red phase. To increase the capacity of a highway, one should find the way to limit the maximum value of the inflow fluctuations. As the downstream flow increases, the critical value Δq_c decreases. The congestion is easier to induced and the highway becomes more unstable. On the other hand, if one could limit the value of Δq_c, the restriction can be pushed toward the limit of $q_{up}+q_{rmp}<2300$ veh/h.

The above control scheme based on the phase diagram is a conservative one. The highway traffic is maintained at the status where the congestion is impossible to emerge. The traffic flow is kept under a certain upper bound. Thus, the capacity of a highway is limited. From the study of the density profile, we propose a new control scheme to eliminate the congestion more effectively. The small traffic jam propagates downstream with a decreasing speed can be taken as an early symptom of the later emerged congestion. By closing the on ramp for a short period, say 30 s, the small traffic jam can be totally eliminated. Thus, the congestion will not appear. The typical results are shown in Fig. 6. In this scheme, the timing is important. When the traffic jam is still developing and propagates downstream, it can be eliminated by closing the on ramp shortly. When the traffic jam is well developed and starts to propagate upstream, it can only be eliminated by closing the ramp for a much longer period, say 10 min. Once the congestion appeared at the ramp, closing the ramp will no longer be an option. At this stage, usually it will take more than half an hour to dissipate the congestion. In this scheme, closing the on ramp temporarily provides an effective way to suppress the formation of congestion. The on ramp is open until the symptom of congestion appeared. Then the ramp is closed for a short period. With the advance of intelligent transportation systems, the highway traffic will be monitored more carefully. Thus makes the scheme possible. Whenever a small traffic jam propagates downstream with a decreasing speed is detected, closing the on ramp temporarily will keep the highway free from congestion in a later time.

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$$\tau = 0.5 \text{ min},$$

$$c_0 = 54 \text{ km/h},$$

$$\mu = 600 \text{ veh km/h},$$

$$V(\rho) = V_0 \frac{1-\rho/\rho_0}{1+100(\rho/\rho_0)^4},$$

$$\phi(x) = \frac{1}{\sqrt{2\pi}\ \sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right),$$

where $V_0 = 120$ km/h and $\rho_0 = 140$ veh/km are the maximum values for velocity and density, respectively, and $\sigma = 60$ m denotes the length of the on ramp.
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